Find a 2 by 3 system $Ax = b$ whose complete solution is

$$x = \begin{bmatrix}1\\2\\0\end{bmatrix} + w\begin{bmatrix}1\\3\\1\end{bmatrix}.$$

$$\begin{bmatrix}1 & -\frac{2}{3} & 1\\ 0 & 1 & -3\end{bmatrix}\begin{bmatrix}x\end{bmatrix}$$

$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}1\\3\\1\end{bmatrix}$$

1) $x_1 + ax_2 + bx_3 = 0$

2) $x_2 + cx_3 = 0$

$x_2 = -cx_3$

$x_2 = -c$

$3 = -c$

$\boxed{c = -3}$

$1 + 3a + b = 0$

if $b = 1$

$2 + 3a = 0 \Rightarrow a = -\frac{2}{3}$

$$\begin{bmatrix}1 & a & b\\ 0 & 1 & -3\end{bmatrix}\begin{bmatrix}y\end{bmatrix} = z \qquad \begin{bmatrix}y\end{bmatrix} = \begin{bmatrix}1\\8\end{bmatrix}$$

$1 + 2a = z_1$ $\Rightarrow$ $1 + 2(-\frac{2}{3}) = z_1$

$2 = z_2$ $\qquad 1 - \frac{4}{3} = z_1$

$$\begin{bmatrix}1 & -\frac{2}{3} & 1\\ 0 & 1 & -3\end{bmatrix}\begin{bmatrix}x\end{bmatrix} = \begin{bmatrix}-\frac{1}{3}\\2\end{bmatrix} \qquad z_1 = -\frac{1}{3}$$

$a - \frac{2}{3}b + c = 0$

$b - 3c = 0$

if $c = 1$

$b = 3$

$a - 2 + 1 =$

$a = 1$

$$N(A) = \begin{bmatrix}1\\3\\1\end{bmatrix}w$$

$$\begin{bmatrix}1 & 0 & a\\ 0 & 1 & b\end{bmatrix}\begin{bmatrix}1\\3\\1\end{bmatrix} = 0$$

$a = -1 \quad b = -3$

$$\begin{bmatrix}1 & 0 & -1\\ 0 & 1 & -3\end{bmatrix}\begin{bmatrix}1\\2\\0\end{bmatrix} = \begin{bmatrix}1\\2\end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & -1\\ 0 & 1 & -3\end{bmatrix}x = \begin{bmatrix}1\\2\end{bmatrix}$$

Linear independence

2E Suppose $c_1 v_1 + \cdots + c_k v_k = 0$ only happens when $c_1 = \cdots = c_k = 0$. Then the vectors $v_1, \ldots, v_k$ are ***linearly independent***. If any c's are nonzero, the v's are ***linearly dependent***. One vector is a combination of the others.

$$\begin{bmatrix} a & c & e \\ b & d & f \end{bmatrix}$$

$$c_1 \begin{bmatrix} v_1 \end{bmatrix} + c_2 \begin{bmatrix} v_2 \end{bmatrix} + c_3 \begin{bmatrix} v_3 \end{bmatrix} = 0$$

if at least one of $c_1 \cdots c_3$ wasn't $= 0$.

→ linearly dependent.

① if $c_1, c_2, c_3 \neq 0$

$$c_1 v_1 + c_2 v_2 = -c_3 v_3$$

② if $c_1, c_2 \neq 0$

$$c_1 v_1 = -c_2 v_2$$

③ if $c_1 \neq 0$

$$c_1 \boxed{v_1} = 0$$

$\to 0$

$$2x + 3y = 10$$

$$4x + 6y = 10$$

$$\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} \to \begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \boxed{2} \\ 2 & -1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

$-1(C_1) + C_2$

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & -4 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & -4 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 5 \\ -1 & -3 & 3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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